10. Solidification Time in Casting

A cylinder with a diameter-to-height ratio of 1 solidifies in 4 min during a sand-casting operation.

1. What will be the solidification time if the cylinder height is tripled?

From Chvorinov's rule and assuming $n = 2$, we have:

$$t = C\left(\frac{V}{A}\right)^2 = C\left[\frac{\frac{\pi d^2 h}{4}}{\frac{\pi d^2}{2} + \pi d h}\right]^2 = C\left(\frac{dh}{2d + 4h}\right)^2 = 4\ min$$

Solving for C:

$$C = 4\ [min]\left(\frac{2d + 4h}{dh}\right)^2 = 4\ [min]\left(\frac{\frac{2d}{h} + 4}{d}\right)^2$$

With the ratio $d/h = 1$:

$$C = 144\ [min]\left(\frac{1}{d}\right)^2$$

If the height is tripled, we can use $d_2 = d$ and $h_2 = 3h$ to obtain the solidification time for the new condition as:

$$t = C\left(\frac{d_2 h_2}{2d_2 + 4h_2}\right)^2 = 144\ [min]\left(\frac{1}{d}\right)^2\left(\frac{3d}{\frac{2d}{h} + 12}\right)^2\Bigg|_{d/h=1}$$

$$t = 144\ [min]\left(\frac{3}{2 + 12}\right)^2 \cong 6.6\ min$$

2. What will be the solidification time if the cylinder diameter is tripled?

If the diameter is tripled, so that $d_3 = 3d$ and $h_3 = h$, then:

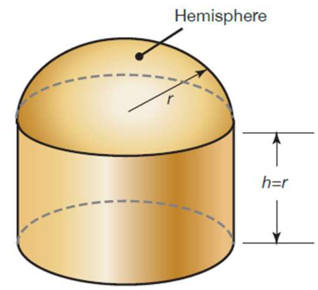
$$t = C\left(\frac{d_3 h_3}{2d_3 + 4h_3}\right)^2 = 144\ [min]\left(\frac{1}{d}\right)^2\left(\frac{3dh}{2(3d) + 4h}\right)^2 = 144\ [min]\left(\frac{1}{d}\right)^2\left(\frac{3d}{\frac{6d}{h} + 4}\right)^2\Bigg|_{d/h=1}$$

$$t = 144\ [min]\left(\frac{3}{6 + 4}\right)^2 \cong 13\ min$$

3. In turns out that the optimum shape of a riser is spherical to ensure that it cools more slowly than the related casting. Spherically shaped risers, however, are difficult to cast.
   a. Sketch the shape of a blind riser that is easy to mold, but also has the smallest possible surface area-to-volume ratio.
   Note. A closed or blind riser is a riser that is contained within the mold. It has no direct contact with the atmosphere.

A sketch of a blind riser that is easy to cast is shown below, consisting of cylindrical and a hemispherical portion.

Note that the height of the cylindrical portion is equal to its radius (so that the total height of the riser is equal to its diameter).



b. Compare the solidification time of the riser you designed to that of a riser shaped like a simple cylinder. Assume that the volume of each riser is the same ($V = V_s$, where V is the volume of a cylindrical riser with height equal to its radius and V_s the volume your optimized riser).
Hint. Consider an arbitrary unity volume, $V = V_s = 1$ and remember that for both the radius-to-cylinder height ratio is equal to 1.

The volume V_s of this optimized, hemispherical riser is

$$V_s = \pi r^2 h + \frac{1}{2}\left(\frac{4\pi r^3}{3}\right) = \frac{5\pi r^3}{3}$$

Letting V_s be unity, we have

$$r = \left(\frac{3}{5\pi}\right)^{1/3}$$

The surface area of the optimized riser is

$$A_s = 2\pi r h + \pi r^2 + \frac{1}{2}(4\pi r^2) = 5\pi r^2$$

Substituting for r, we find $A_s \cong 5.21$. Therefore, the solidification time for the blind riser will be

$$t_s = B\left(\frac{V_s}{A_s}\right)^2 = B\left(\frac{1}{5.21}\right)^2 \cong 0.037B$$

For a purely cylindrical riser of the same volume, we have

$$V = \pi r'^2 h' = \pi r'^3 \equiv 1$$

$$r' = \left(\frac{1}{\pi}\right)^{1/3}$$

$$A = 2\pi r'^2 + 2\pi r' h' = 2\pi r'^2 + 4\pi r'^2 = 6\pi r'^2 \cong 8.79$$

$$t = B\left(\frac{V}{A}\right)^2 = B\left(\frac{1}{8.79}\right)^2 \cong 0.013B$$

Thus, the optimized blind riser with hemispherical shape will cool three times slower while being easily casted (at least way easier than a fully spherical riser).

11. Turning a Ti-Alloy Rod

A titanium-alloy rod of length $l = 150$ mm and diameter $\emptyset_i = 75$ mm is being radially reduced to a diameter $\emptyset_f = 65$ mm by turning on a lathe in one pass. The spindle rotates at $v_s = 400$ rpm and the tool is traveling at an axial velocity of $v_a = 200$ mm/min. Consider that the unit energy required has an average value of $E_{av} = 3.5\ \mathrm{W \cdot s/mm^3}$.

1. Calculate the material removal rate.

The depth of cut can be calculated from the information given as

$$d = \frac{\emptyset_i - \emptyset_f}{2} = \frac{75 - 65}{2} = 5\ \mathrm{mm}$$

and the average diameter is $D_{av} = \frac{75+65}{2} = 70$ mm.

Moreover, the feed is

$$f = \frac{v_a}{v_s} = \frac{200}{400} = 0.50\ \mathrm{mm/rev}$$

Therefore, we can compute

$$MRR = \pi D_{av} d f v_s = \pi \cdot 70 \cdot 5 \cdot 0.50 \cdot 400 = 2.2 \cdot 10^5\ \mathrm{mm^3/min}$$

2. How long does it take for this machining operation to be completed?

The actual time to cut is given by

$$t = \frac{l}{v_a} = \frac{150}{200} = 0.75\ min = 45\ \mathrm{s}$$

3. What is the required power?

$$P = E_{av} \cdot MRR = \frac{3.5 \cdot 2.2 \cdot 10^5}{60} \cong 13\ \mathrm{kW}$$

4. What is the loading mode in the rod? Characterize and calculate the cutting force F_c.

The material is subjected to pure shear stress, and the cutting force is the tangential force exerted by the tool.

The spindle speed is $v_s = 400$ rpm $\equiv \omega = 41.89$ rad/s. Since $P = T \cdot \omega$, we have:

$$T = \frac{P}{\omega} = \frac{13 \cdot 10^3}{41.89} \cong 310\ \mathrm{N \cdot m}$$

Dividing the torque by the average workpiece radius, we have

$$F_c = \frac{T}{\frac{D_{av}}{2}} = \frac{310}{0.035} \cong 8.9\ \mathrm{kN}$$

12. Tool Wear with Ceramics

Using the Taylor equation for tool wear and choosing the average value out of the possible n values for tools in ceramics, calculate the percentage increase in tool life if the cutting speed is reduced by (a) 30% and (b) 60%.

The Taylor equation for tool wear is an empirical model. It was given in the lecture on Conventional Machining (as well as n for different materials) and can be rewritten for the average case of ceramics as

$$C = V \cdot LT^n = V \cdot LT^{0.6}$$

(where $V = w^x t_0^y v$).

We can compare to cases as

$$V_1 LT_1^n = V_2 LT_2^n \Rightarrow \frac{LT_1}{LT_2} = \left(\frac{V_2}{V_1}\right)^{1/n}$$

(a) For the case where the speed is reduced by 30%, then $V_2 = 0.7V_1$, and thus

$$\frac{LT_1}{LT_2} = \left(\frac{0.7V_1}{V_1}\right)^{1/0.6} \cong 0.55$$

The new life LT_2 is 1.8 times the initial lifetime.

(b) For the case where the speed is reduced by 60%, then $V_2 = 0.4V_1$, and thus

$$\frac{LT_1}{LT_2} = \left(\frac{0.4V_1}{V_1}\right)^{1/0.6} \cong 0.22$$

The new life LT_2 is 4.5 times the initial lifetime.

13. Drilling Holes in a Block of Magnesium

(Adapted from the final exam 2018)

A hole is being drilled in a block of magnesium alloy of thickness $t_b = 2$ cm, with a $\emptyset_d = 15$ mm drill in high-speed steel ($\sigma_{y,HSS} = 1000$ MPa, $E_{HSS} = 200$ GPa, $\nu_{HSS} = 0.29$) at a feed of $f = 0.20$ mm/rev. The spindle is running at $v_s = 500$ rpm.
Consider that the unit energy required has an average value of $E_{av} = 0.5\ \mathrm{W \cdot s/mm^3}$.

1. Express the MRR for a drill bit of diameter $\emptyset_d$, a feed rate f and spindle rotational speed v_s.

The MRR can be calculated logically from the volume of matter removed per turn and at a certain rate:

$$MRR = \frac{\pi \emptyset_d^2}{4} f v_s$$

2. Calculate the MRR, and estimate the torque on the drill.

$$MRR = \frac{\pi 15^3}{4} \cdot 0.20 \cdot 500 \cong 1.8 \cdot 10^4\ \mathrm{mm^3/min} \cong 300\ \mathrm{mm^3/s}$$

Using the average specific energy for magnesium alloys, we find:

$$P = E_{av} \cdot MRR = 0.5 \cdot 300 = 150\ \mathrm{W}$$

The power is the product of the torque on the drill and the rotation speed in radians per second, which, in this case is:

$$\omega = \frac{2\pi v_s}{60} = \frac{2\pi \cdot 500}{60} \cong 52.4\ \mathrm{rad/s}$$

Which gives for the torque:

$$T = \frac{P}{\omega} = \frac{150}{52.4} \cong 2.86\ \mathrm{N \cdot m}$$

3. Express and compute the angle of twist ϕ seen by the drill during the process as a function of the given parameters.

The maximum shear strain on the drill is $\gamma = \frac{\emptyset_d \phi}{L}$ where ϕ is the angle of twist and $L \geq t_b$ is the drill length.

The maximum shear stress on the drill is $\tau = \frac{\emptyset_d T}{J}$ where $J = \frac{\pi}{2}\left(\frac{\emptyset_d}{2}\right)^4$ is the second moment of area of the drill (assuming that the drill is a solid rod).

Now we can use Hooke's law in shear for the angle of twist:

$$\tau = G\gamma = G\frac{\emptyset_d \phi}{L} = \frac{\emptyset_d T}{J} \Rightarrow \phi = \frac{TL}{GJ} = \frac{64(1+\nu)}{\pi}\frac{TL}{E}\left(\frac{1}{\emptyset_d}\right)^4$$

Numerical application:

$$\phi = \frac{64(1+0.29)}{\pi}\frac{2.86 \cdot 2 \cdot 10^{-2}}{200 \cdot 10^9}\left(\frac{1}{15 \cdot 10^{-3}}\right)^4 \cong 1.48 \cdot 10^{-4}\ \mathrm{rad} \cong 0.00850°$$